University of Dayton

Homework assignment 1

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AEE 401/501-Aerodynamics 1

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<u>1.1.</u>

Find:

a) ρ where

$$R = 287\frac{J}{kg.K} = 1716\frac{ft.Ib}{slug.°R}, \qquad p = 1.9\times 10^4\frac{N}{m^2}, \qquad T = 203K$$

b) T where

$$R = 1716\frac{ft.Ib}{slug.°R}, \qquad p = 1058\frac{Ib}{ft^2}, \qquad \rho = 1.23\times 10^{-3}\frac{slug}{ft^3}$$

<u>Solution:</u>

a)

$$\boxed{p = \rho RT \Longrightarrow \rho = \frac{p}{RT}}$$

$$\therefore \rho = \frac{p}{RT} = \frac{1.9\times 10^4\frac{N}{m^2}}{287\frac{N.m}{kg.K}\times 203K} = 0.3261\frac{kg}{m^3}$$

b)

$$\boxed{p = \rho RT \Longrightarrow T = \frac{p}{\rho R}}$$

$$\therefore T = \frac{p}{\rho T} = \frac{1058\frac{Ib}{ft^2}}{1.23\times 10^{-3}\frac{slug}{ft^3}\times 1716\frac{ft.Ib}{slug.°R}} = 501.26\frac{kg}{m^3}$$

1.3

Given: flat plate of chord c at an angle of attack α in a supersonic flow as shown below;

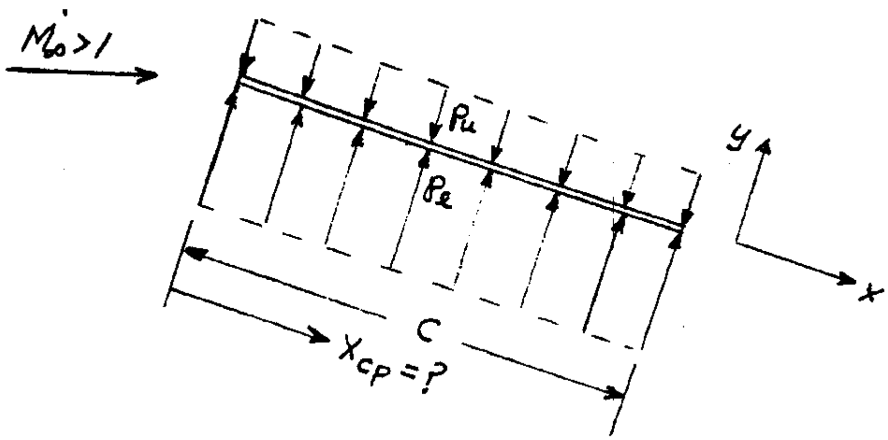

Figure 1: schematic diagram of problem 1.3.

where

$$\tau = 0, \qquad \theta = 0(Flat\ plate)$$

Find:

$$x_{cp} = ?$$

Solution:

From equation 1.20, we have

$$\boxed{x_{cp} = -\frac{M_{LE}}{N}} \tag{1.20}$$

The general equation of the moment about the leading edge per unit span is;

$$\boxed{\begin{aligned} M_{LE} = &\int_{LE}^{TE} [(p_u \cos\theta + \tau_u \sin\theta)x - (p_u \sin\theta - \tau_u \cos\theta)y]ds_u + \\ &\int_{LE}^{TE} [(-p_l \cos\theta + \tau_l \sin\theta)x + (p_l \sin\theta + \tau_l \cos\theta)y]ds_l + \end{aligned}} \tag{1.11}$$

The general form of the total normal force per unit is given as follows;

$$\boxed{N = -\int_{LE}^{TE} (p_u \cos\theta + \tau_u \sin\theta)ds_u + \int_{LE}^{TE} (p_l \cos\theta - \tau_l \sin\theta)ds_l} \tag{1.7}$$

Since we know that $\tau = 0$ and for a flat plate $\theta = 0$. Therefore, equation (1.11) and (1.7) can be simplified as follows;

$$M_{LE} = \int_{LE}^{TE} [(p_u)x]ds_u + \int_{LE}^{TE} [(-p_l)x]ds_l \qquad (1.11.a)$$

It is given in the problem that the pressure on both sides is constant. Hence,

$$M_{LE} = (p_u - p_l) \int_{LE}^{TE} s ds = (p_u - p_l) \int_{0}^{c} s ds \qquad (1.11.b)$$

$$\boxed{M_{LE} = (p_u - p_l)\frac{c^2}{2}} \qquad (1.11.c)$$

Similarly, for the normal force equation (1.7);

$$N = -\int_{LE}^{TE} (p_u)ds_u + \int_{LE}^{TE} (p_l)ds_l \qquad (1.7.a)$$

$$N = -(p_u - p_l) \int_{LE}^{TE} ds = -(p_u - p_l) \int_{0}^{c} ds \qquad (1.7.b)$$

$$\boxed{N = -(p_u - p_l)c} \qquad (1.7.c)$$

Substituting equation (1.7.c) and (1.11.c) into (1.20) yields;

$$\boxed{x_{cp} = -\frac{(p_u - p_l)\frac{c^2}{2}}{-(p_u - p_l)c} = \frac{c}{2}} \qquad (\#)$$

1.9.

Solution:

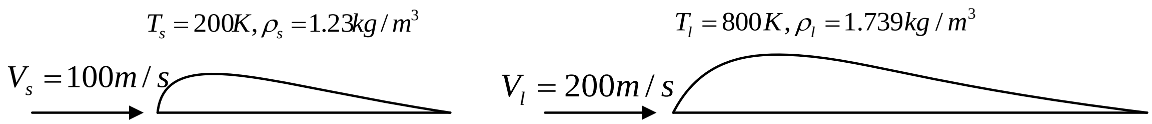

Figure1: shows the airfoils

Whether the two flows dynamically similar or not requires from us to check many factors. In addition to that, they have to have the same drag and lift coefficients. The similarity parameters are Mach number and Reynolds's number.

Mach number:

$$\frac{M_s}{M_l} = \frac{\frac{V_s}{a_s}}{\frac{a_l}{V_l}} = \frac{\frac{V_s}{\sqrt{\gamma R T_s}}}{\frac{a_l}{\sqrt{\gamma R T_l}}} = \frac{\frac{V_s}{\sqrt{T_s}}}{\frac{a_l}{\sqrt{T_l}}} = \frac{V_s}{V_l}\sqrt{\frac{T_l}{T_s}} = \frac{100}{200}\sqrt{\frac{800}{200}} = 1$$

Mach Numbers are the same.

Reynolds's number:

$$\frac{Re_s}{Re_l} = \frac{\frac{\rho_s V_s c_s}{\mu_s}}{\frac{\rho_l V_l c_l}{\mu_l}} = \frac{\rho_s V_s c_s}{\rho_l V_l c_l}\frac{\mu_l}{\mu_s}$$

Sutherland law:

$$\mu(T) = \mu_{ref}\left(\frac{T}{T_0}\right)^{3/2}\frac{T_0 + S}{T + S}, \qquad S_{air} = 111K$$

Substituting Sutherland equation for the dynamic viscosity into Reynolds's number produces:

$$\frac{Re_s}{Re_l} = \frac{\rho_s V_s c_s}{\rho_l V_l c_l}\frac{\mu_{ref}\left(\frac{T_l}{T_0}\right)^{3/2}\frac{T_0 + S}{T_l + S}}{\mu_{ref}\left(\frac{T_s}{T_0}\right)^{3/2}\frac{T_0 + S}{T_s + S}} \Longrightarrow \frac{Re_s}{Re_l} = \frac{\rho_s V_s c_s}{\rho_l V_l c_l}\left(\frac{T_l}{T_s}\right)^{3/2}\frac{(T_s + S)}{(T_l + S)}$$

$$\therefore \frac{Re_s}{Re_l} = \frac{1.23 \times 100 \times 1}{1.739 \times 200 \times 2}\left(\frac{800}{200}\right)^{3/2}\frac{(200 + 111)}{(800 + 111)}$$

$$\frac{Re_s}{Re_l} = \frac{1.23 \times 100 \times 1}{1.739 \times 200 \times 2} \left(\frac{800}{200}\right)^{3/2} \frac{(200 + 111)}{(800 + 111)} \neq 1$$

Since $Re \neq 1$, therefore, the two flows are not dynamically similar.

1.6

$\alpha(degree)$	C_l	C_d	$C_{m,c/4}$
-2	0.05	0.006	-0.042
0	0.25	0.006	-0.040
2	0.44	0.006	-0.038
4	0.64	0.007	-0.036
6	0.85	0.0075	-0.036
8	1.08	0.0092	-0.036
10	1.26	0.0115	-0.034
12	1.43	0.0150	-0.030
14	1.56	0.0186	-0.026

From this table, plot the variation of x_{cp}/c as a function of α. Comment on the result.

Solution:

Using equation (1.22), we derive for the airfoil that has the quarter chord the following equation:

$$x_{cp} = \frac{c}{4} - \frac{M'_{c/4}}{N'}$$

Also, by assuming that $N' \approx L' \Longrightarrow c_n \approx c_l$, we get:

$$x_{cp} = \frac{c}{4} - \frac{M'_{c/4}}{N'} = \frac{c}{4} - \left(\frac{c_{M_{c/4}}}{c_n}\right)$$

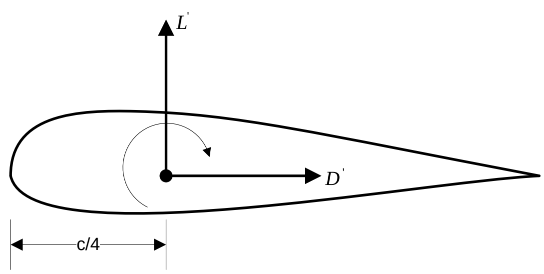

Figure 2

From equations (1.18) and (1.19)

$$c_n = c_l \cos\alpha - c_a \sin\alpha \qquad (1.18)$$

$$c_d = c_n \sin\alpha + c_a \cos\alpha \qquad (1.19)$$

We can drive the following equation:

From equation (1.18), we can rearrange it to be:

$$c_a = \frac{c_n \cos\alpha - c_l}{\sin\alpha} \tag{1.18a}$$

Substituting equation (1.18a) into equation (1.19) produces:

$$c_d = c_n \sin\alpha + \left(\frac{c_n \cos\alpha - c_l}{\sin\alpha}\right)\cos\alpha$$

Multiplying both sides of the equation by $\sin\alpha$ produces:

$$c_d \sin\alpha = \underbrace{c_n \sin^2\alpha + c_n \cos^2\alpha}_{\sin^2\alpha+\cos^2\alpha=1} - c_l \cos\alpha$$

$$\boxed{\therefore\ c_n = c_l \cos\alpha + c_d \sin\alpha}$$

$\alpha(degree)$	C_l	C_d	$C_{m,c/4}$	C_n	x_{cp}/c
-2	0.05	0.006	-0.042	0.04976	1.094051447
0	0.25	0.006	-0.040	0.25	0.41
2	0.44	0.006	-0.038	0.439941	0.336375218
4	0.64	0.007	-0.036	0.638685	0.306365814
6	0.85	0.0075	-0.036	0.846124	0.292546955
8	1.08	0.0092	-0.036	1.07078	0.283620352
10	1.26	0.0115	-0.034	1.24286	0.277356259
12	1.43	0.0150	-0.030	1.40321	0.271379551
14	1.56	0.0186	-0.026	1.51816	0.267125995

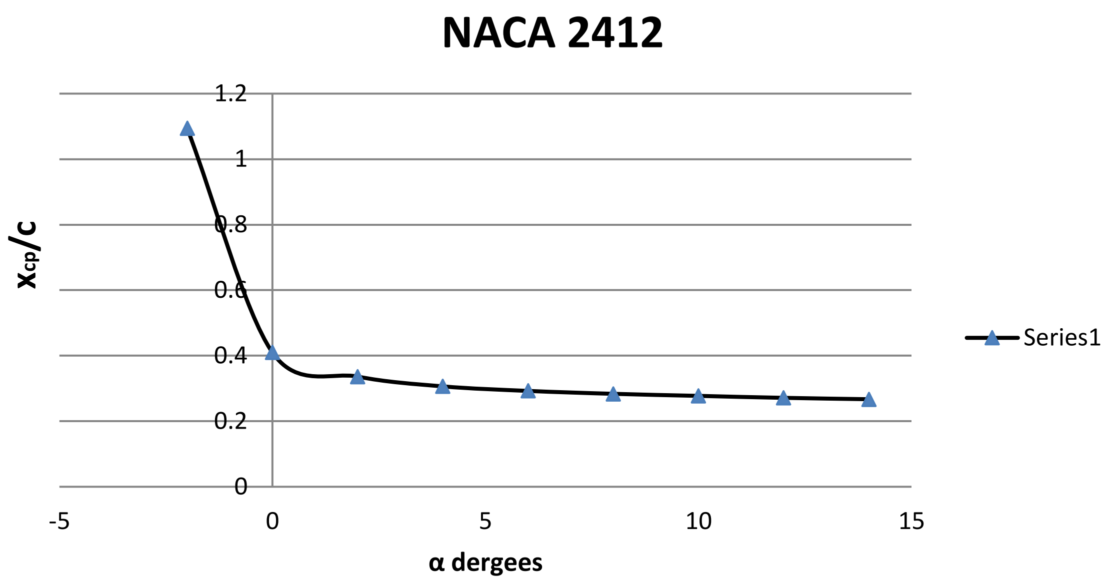

Figure 3

From figure 3, the data between x_{cp}/c and α shows that at the approximate angle 0° the increase of the angle of attack makes x_{cp}/c stay almost constant.